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A NEW DEPLOYMENT/RETRIEVAL SCHEME FOR A TETHERED SATELLITE SYSTEM, INTERMEDIATE BETWEEN THE CONVENTIONAL SCHEME AND THE CRAWLER SCHEME†

M. PASCAL, A. DJEBLI and L. EL BAKKALI

Paris, France

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The dynamics of a tethered satellite system (TSS) during deployment and retrieval in orbit are considered. The system consists of a space station and a second body (a sub-satellite) connected to it by a tether. The station and sub-satellite are assumed to be point masses whose mass centre describes an unperturbed Kepler orbit. Deployment/retrieval schemes are defined in which the ordinary differential equations with variable coefficients that describe small spatial oscillations of the system can be solved by analytical means. An intermediate scheme, generalizing the previously proposed conventional scheme and the crawler scheme [1] and which has certain advantages, is obtained.

The use of a tethered satellite system (TSS) has been proposed for solving a large number of space dynamics problems, such as guaranteeing the return of astronauts working in free space, the construction of long antennas, the performance of experiments in the upper atmosphere, etc. [2]. Our main attention in this paper is devoted to investigating the oscillations of a system consisting of a space station and a second body (a sub-satellite) connected to it by a tether, during deployment and retrieval. Such a system may experience large vibrations which destabilize the whole system [3, 4]. In addition to the linear and exponential laws considered previously for the variation of the tether length, new laws have been proposed [1]; the nature of the oscillations developed under these laws has been compared, and in addition to the conventional scheme a crawler scheme has been proposed, in which the sub-satellite crawls along a tether deployed in advance for its whole length.

The aim of this paper is to investigate the behaviour of a system with an intermediate deployment/retrieval scheme, which generalizes the conventional and crawler schemes.

1. FORMULATION OF THE PROBLEM

Consider a TSS consisting of a space station and a second body (a sub-satellite) moving along a straight tether of variable length $L(t)$. The space station and sub-satellite are assumed to be point masses M_1 and M_2 of masses m_1 and m_2 and $S(t) \leq L(t)$ is the distance between the points M_1 and M_2. Assuming that the tether is deployed from the space station, we will assume that the mass m_2 of the sub-satellite is constant, while the masses m_1 and m_3 of the space station and tether depend on time. This leads to the relations

$$\mu_1 + \mu_2 + \mu_3 = 1, \quad \dot{\mu}_1 + \dot{\mu}_3 = 0, \quad \dot{\mu}_3 = \mu_3 \dot{L} / L \tag{1.1}$$

$$m_3 = \rho L, \quad \mu_i = m_i / m, \quad i = 1, 2, 3$$

where ρ is the density of the tether and m is the (constant) total mass of the system. We shall assume that the mass centre G of the system is moving in the equatorial plane of Earth. We introduce polar coordinates ν and R ($R = OG$, where O is Earth's centre) in the orbital plane. Two angle θ and φ define the angular position of the tether in the orbital coordinate system $Gxyz$ (y is the local vertical and x the normal to the orbital plane (Fig. 1)). The kinetic energy of the system is

$$T = \frac{m}{2}\{R^2 + S^2 A[(\dot{\theta} + \dot{\nu})^2 \cos^2 \varphi + \dot{\varphi}^2] + B\dot{S}^2 + \mu_3[(1 - \mu_3)\dot{L} - 2\mu_2 \dot{S}]\} \tag{1.2}$$

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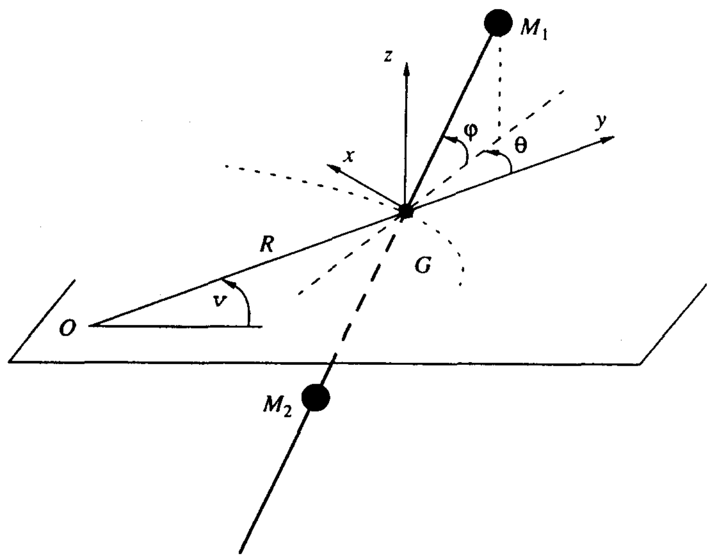

Fig. 1

where

$$A=\mu_1\mu_2+\frac{\mu_3}{4}\left(\mu_1+\mu_2+\frac{1}{3}\right)\left(\frac{L}{S}\right)^2-\mu_2\mu_3\left(\frac{L}{S}-1\right),\quad B=\mu_2(\mu_1+\mu_3) \tag{1.3}$$

The potential energy of the gravitational forces in the satellite approximation is (f is the gravitational constant)

$$V=-\frac{fm}{R}\left[1+\frac{S^2A}{2R^2}(3\cos^2\theta\cos^2\varphi-1)\right] \tag{1.4}$$

The kinetic and potential energies [1] of the system with the conventional deployment/retrieval scheme and the crawler scheme may clearly be obtained from formulae (1.2)–(1.4) by setting $L(t) = S(t)$ for the conventional scheme and L = const for the crawler scheme.

The equations of motion of the system with the intermediate scheme will be written in the form of Lagrange's equations

$$\frac{d}{dt}\frac{dT}{\partial\dot{q}_i}-\frac{\partial(T-V)}{\partial q_i}=Q_i,\quad q_i=(R,\nu,S,L,\theta,\varphi)^T \tag{1.5}$$

where Q_i are generalized non-potential forces (attitude-control forces, air drag, etc.).

Let us assume that the mass centre of the system is moving in an elliptical orbit:

$$R=\frac{h^2}{f(1+e\cos\nu)},\quad R^2\dot{\nu}=h\quad (h,e=\text{const},\ e<1) \tag{1.6}$$

and introduce the following notation (throughout, the prime will denote differentiation with respect to ν)

$$\sigma=1+e\cos\nu,\quad \tilde{G}=\frac{\tilde{p}'}{\tilde{p}},\quad \tilde{p}=\frac{\tilde{S}}{\tilde{S}(0)},\quad \tilde{S}(\nu)=S\sigma\sqrt{A} \tag{1.7}$$

$$\tilde{Q}_\theta=\frac{Q_\theta}{mS^2A\dot{\nu}^2},\quad \tilde{Q}_\varphi=\frac{Q_\varphi}{mS^2A\dot{\nu}^2}$$

Then the spatial angular oscillations of the tether will be defined by equations

$$\theta''\cos^2\varphi + 2\tilde{G}(\theta'+1)\cos^2\varphi - \varphi'(\theta'+1)\sin(2\varphi) + \frac{3}{2\sigma}\cos^2\varphi\sin(2\theta) = \tilde{Q}_\theta \tag{1.8}$$

$$\varphi'' + 2\tilde{G}\varphi' + \frac{(\theta'+1)^2}{2}\sin(2\varphi) + \frac{3}{2\sigma}\cos^2\theta\sin(2\varphi) = \tilde{Q}_\varphi$$

2. PARTICULAR DEPLOYMENT/RETRIEVAL LAWS IN A CIRCULAR ORBIT

Assuming that the mass centre G is moving in a circular orbit ($e = 0$) at constant angular velocity ω ($\nu = \omega t$), we obtain the linearized equations of small oscillations of the tether in the neighbourhood of the local vertical from Eqs (1.8) by setting $\theta \ll 1$ and $\varphi \ll 1$:

$$\theta'' + 2\tilde{G}(\theta'+1) + 3\theta = \tilde{Q}_\theta, \quad \varphi'' + 2\tilde{G}\varphi' + 4\varphi = \tilde{Q}_\varphi \tag{2.1}$$

In the intermediate deployment/retrieval scheme, as in the case of the conventional and crawler schemes [1], one can choose special laws governing the variation of the tether length $L(t)$ and the displacement of the sub-satellite $S(t)$ along the tether, so as to obtain an analytical solution of system (2.1).

Let $\tilde{G}$ be á solution of the following differential equation ($\delta < 3$ is an arbitrary constant):

$$\tilde{G}' + \tilde{G}^2 = \delta \tag{2.2}$$

Then small spatial oscillations of the tether are defined by the formulae

$$\theta = -\frac{2}{3}\tilde{G} + \frac{1}{\tilde{p}}\left[C\sin(\delta_\theta\nu + \varphi_1) + \frac{1}{\delta_\theta}\int_0^\nu \sin(\delta_\theta(\nu-\tau))\hat{Q}_\theta(\tau)d\tau\right]$$

$$\varphi = \frac{1}{\tilde{p}}\left[D\sin(\delta_\varphi\nu + \varphi_2) + \frac{1}{\delta_\varphi}\int_0^\nu \sin(\delta_\varphi(\nu-\tau))\hat{Q}_\varphi(\tau)d\tau\right] \tag{2.3}$$

$$\delta_\theta = \sqrt{3-\delta}, \quad \delta_\varphi = \sqrt{4-\delta}, \quad \hat{Q}_\theta = \tilde{p}\tilde{Q}_\theta, \quad \hat{Q}_\varphi = \tilde{p}\hat{Q}_\varphi$$

where C, D, φ_1 and φ_2 are arbitrary constants. From Eq. (2.2) we obtain the following differential equation for $\tilde{p}(\nu)$

$$\tilde{p}'' - \delta\tilde{p} = 0, \quad \tilde{p}(0) = 1 \tag{2.4}$$

As in the cases of the conventional and crawler schemes, we choose some special forms of $\tilde{p}(\nu)$ depending on the sign of δ (hyperbolic, exponential, linear sinusoidal relations

$$\delta > 0, \quad \tilde{p}_{\rm h}y_{\rm p} = \frac{\operatorname{sh}(\nu_f\sqrt{\delta}(1-\tau)) + \tilde{p}_f\operatorname{sh}(\nu_f\sqrt{\delta}\tau)}{\operatorname{sh}(\nu_f\sqrt{\delta})}$$

$$\delta = \tilde{G}^2(0), \quad \tilde{p}_{\rm exp} = \tilde{p}_f^\tau$$

$$\delta = 0, \quad \tilde{p}_{\rm lin} = 1 + (\tilde{p}_f - 1)\tau \tag{2.5}$$

$$\delta < 0, \quad \tilde{p}_{\rm sin} = \frac{\sin(\nu_f\sqrt{-\delta}(1-\tau)) + \tilde{p}_f\sin(\nu_f\sqrt{-\delta}\tau)}{\sin(\nu_f\sqrt{-\delta})}$$

$$\tilde{p}_f = Xp_f, \quad p_f = S_f/S_0, \quad X = \sqrt{A_f/A_0}$$

$$\tau = \nu/\nu_f, \quad \nu_f = \omega t_f$$

where t_f is the total deployment or retrieval time from the initial state S_0, L_0 to the final state S_f, L_f; A_0 and A_f are evaluated by formula (1.3) for S_0, L_0 and S_f, L_f, respectively.

The initial values of $\tilde{G}$ are obtained from (2.5)

$$\tilde{G}_0 = \begin{cases} \sqrt{\delta}\,\dfrac{\tilde{p}_f - \mathrm{ch}(\nu_f\sqrt{\delta})}{\mathrm{sh}(\nu_f\sqrt{\delta})}, & \delta > 0 \\ \dfrac{\tilde{p}_f - 1}{\nu_f}, & \delta = 0 \\ \sqrt{-\delta}\,\dfrac{\tilde{p}_f - \cos(\nu_f\sqrt{-\delta})}{\sin(\nu_f\sqrt{-\delta})}, & \delta < 0 \end{cases} \tag{2.6}$$

Hence it follows that, for fixed values of ν_f, the value of $\tilde{G}_0$ is proportional to the final value $\tilde{p}_f$ of $\tilde{p}$.

As before [1], for a monotone variation of $\tilde{p}$ the constant δ must be chosen in the range $[\delta_1^+, \delta_2^+]$ for deployment ($\tilde{p}_f > 1$) and in the range $[\delta_1^-, \delta_2^-]$ for retrieval ($\tilde{p}_f < 1$), where

$$\delta_1^\pm = -\left[\frac{1}{\nu_f}\arccos \tilde{p}_f^{\mp 1}\right]^2, \quad \delta_2^\pm = \left[\frac{1}{\nu_f}\mathrm{Ar\,ch}\,\tilde{p}_f^{\pm 1}\right]^2 \tag{2.7}$$

Using formulae (2.5) for $\tilde{p}$, we can derive corresponding formulae for the variation of the tether length $q(t) = L(t)/L_0$ and the displacement $p(t) = S(t)/S_0$ of the sub-satellite, using the equation

$$Ap^2 - A_0\tilde{p}^2 = 0 \tag{2.8}$$

Introducing the notation

$$K = \frac{S_0}{L_0}, \quad \mu_3^0 = \frac{\rho L_0}{m}$$

$$\alpha = \mu_2(1-\mu_2), \quad \beta(q) = K\mu_2\mu_3^0 q^2, \quad \gamma(q) = K^2\mu_3^0 q^3\left(\frac{1}{3} - \frac{1}{4}\mu_3^0 q\right)$$

we express A and A_0 in the form

$$A = \alpha - \frac{\beta(q)}{p} + \frac{\gamma(q)}{p^2}, \quad A_0 = A\big|_{p=q=1} \tag{2.9}$$

Assuming that $q(t)$, governing the variation of tether length, is known, the displacement $p(t)$ of the sub-satellite along the tether is determined from the equation

$$\alpha p^2 - \beta p + \gamma - A_0\tilde{p}^2 = 0 \tag{2.10}$$

When $\nu = 0$ and $q = 1$, one of the roots of Eq. (2.10) is $p = 1$; when $\nu = \nu_f$, $q = q_f$, one of the roots is $p = p_f$. Hence we obtain the following formulae for the values Δ_0 and Δ_f of the discriminant Δ of Eq. (2.10) at the initial and final times

$$\Delta_0 = [2\mu_2(1-\mu_2) - K\mu_2\mu_3^0]^2 \tag{2.11}$$

$$\Delta_f = [2\mu_2(1-\mu_2)p_f - K\mu_2\mu_3^0 q_f^2]^2$$

In order to obtain an admissible solution of Eq. (2.10), a few additional conditions must be satisfied. For the solution to be real, the discriminant must be positive for $0 \leqslant \nu \leqslant \nu$. In addition, the solution must satisfy the inequality

$$0 < p < Kq \tag{2.12}$$

and, finally, only monotone forms of variation of p are suitable. In the next section, deployment/retrieval schemes for which all these conditions are satisfied will be investigated.

3. NEW DEPLOYMENT/RETRIEVAL SCHEMES

We will investigate a deployment scheme S^+/L^- corresponding to $p_f > 1$, $q_f < 1$ (the distance between the space station and the sub-satellite is increased as the tether is retrieved), and a retrieval scheme S^-/L^+ corresponding to $p_f < 1$, $q_f > 1$ (the distance between the station and the sub-satellite is reduced as the tether is deployed).

In both cases it is assumed that the variations of $\tilde{p}$, p and q are monotone, and the increments to $\tilde{p}$ and p have the same sign. This implies the conditions

$$S^+/L^-: \tilde{p}' > 0, \quad p' > 0, \quad q' < 0 \tag{3.1}$$

$$S^-/L^+: \tilde{p}' < 0, \quad p' < 0, \quad q' > 0$$

Bearing in mind that

$$\mu_3^0 q - 1 + \mu_2 = \mu_3 - 1 + \mu_2 = -\mu_1$$

we obtain the following expression for the derivative of the discriminant of Eq. (2.10)

$$\Delta' = -4K^2\mu_2\mu_3^0 q^2 q'\mu_1 + 8\mu_2(1-\mu_2)A_0\tilde{p}\tilde{p}'$$

Hence it follows that

$$S^+/L^-: \ \Delta' > 0, \quad S^-/L^+: \ \Delta' < 0$$

and, by formulae (2.11), we obtain

$$S^+/L^-: \ 0 < \Delta_0 \leqslant \Delta \leqslant \Delta_f$$

$$S^-/L^+: \ 0 < \Delta_f \leqslant \Delta \leqslant \Delta_0$$

The two solutions of Eq. (2.10) are given by

$$p_\pm = \frac{\beta \pm \sqrt{\Delta}}{2\alpha} \tag{3.2}$$

The solution p_- is not suitable, since it follows from the formula

$$p_-' = \frac{\beta'}{2\alpha} - \frac{\Delta'}{4\alpha\sqrt{\Delta}}, \quad \beta' = 2K\mu_2\mu_3^0 qq' \tag{3.3}$$

that

$$S^+/L^-: \ \tilde{p}' > 0, \quad q' < 0, \quad \Delta' > 0 \Rightarrow p_-' < 0$$

$$S^-/L^+: \ \tilde{p}' < 0, \quad q' > 0, \quad \Delta' < 0 \Rightarrow p_-' > 0$$

Only the solution p_+ remains. We can prove that p_+ varies monotonically provided that the space station has large mass. If $\mu_2 \ll 1$, $\mu_3 \ll 1$, an approximate value of p_+ is

$$p_+ = [\tilde{p}^2(1+b) - bq^3]^{1/2}, \quad b = K^2\frac{\mu_3^0}{3\mu_2} \tag{3.4}$$

For both schemes S^+/L^- and S^-/L^+, the function p_2 is monotone, its increments having the same sign as $\tilde{p}$.

Assuming that $S_0 < L_0$ and $S_f < L_f$, we get

$$S^+/L^-: \ S_0 < S < S_f < L_f < L$$
$$S^-/L^+: \ S_f < S < S_0 < L_0 < L$$

and in both cases $S(\nu) \leqslant L(\nu)$ for $0 \leqslant \nu \leqslant \nu_f$.

4. COMPARISON OF THE NATURE OF OSCILLATIONS SYSTEMS IN DIFFERENT DEPLOYMENT/RETRIEVAL PROCEDURES

For a circular orbit, the spatial motion of the tether will be determined by Eqs (1.8), in which we put $\sigma = 1$, neglecting the quantities $\tilde{Q}_\theta$ and $\tilde{Q}_\varphi$ and also putting $\tilde{G} = \tilde{S}'/\tilde{S}$, $\tilde{S} = S\sqrt{A}$, where A takes different values, depending on the formulae chosen for the variables defining the deployment and retrieval processes:

for the conventional scheme

$$A \equiv A_L = \mu_2(1-\mu_2)+\mu_3\left(\frac{1}{3}-\frac{1}{4}\mu_3\right)-\mu_3\mu_2, \quad \mu_3 = \rho\frac{S(t)}{m} \tag{4.1}$$

for the crawler scheme (L = const, $S(t) \leqslant L$)

$$A \equiv A_S = \mu_2(1-\mu_2)+\mu_3\frac{L^2}{S^2}\left(\frac{4}{3}-\frac{1}{4}\mu_3\right)-\mu_3\mu_2\frac{L}{S}, \quad \mu_3 = \rho\frac{L}{m} \tag{4.2}$$

for the intermediate scheme, A is defined by the first expression in (2.9)

$$A \equiv A_{LS}$$

The behaviour of systems with these deployment/retrieval schemes can be compared qualitatively, by choosing the ratio $X = \tilde{p}_f/p_f$ as the criterion [1].

Equations (1.8) have the particular solution

$$\theta = \theta_0, \quad \theta' = \varphi = \varphi' = 0$$

The constant θ_0 is defined by the formula

$$\sin 2\theta_0 = -\frac{4}{3}\tilde{G} = -\frac{4}{3}\frac{\tilde{S}'}{\tilde{S}} = -\frac{4}{3}\frac{\tilde{p}'}{\tilde{p}} \tag{4.3}$$

For motion at a constant angle of deviation of the deploying tether [5], we obtain from (4.3)

$$\ln \tilde{p}_f = -\frac{3}{4}\nu_f \sin 2\theta_0 \tag{4.4}$$

Assuming that p_f and ν_f have the same values for all three schemes conventional, crawler and intermediate schemes), the resultant deviations of the tether from the local vertical may be compared by using the values obtained for θ_0

$$S^+/L^-: \quad p_f > 1, \quad \tilde{p}_f = Xp_f > 1, \quad |\sin 2\theta_0| = \frac{4}{3}\frac{\ln(Xp_f)}{\nu_f}$$

$$S^-/L^+: \quad p_f < 1, \quad \tilde{p}_f = Xp_f < 1, \quad 0 < \sin 2\theta_0 = -\frac{4}{3}\frac{\ln(Xp_f)}{\nu_f}$$

Hence it follows that, in order to reduce the amplitude of oscillations θ_0 in deployment, the quantity X should be reduced; to achieve the same effect in retrieval, it should be increased.

An estimate of the effect of the tether's mass on the behaviour of the system shows it to be very small [5].

Below we will estimate the dynamic characteristics of the system in deployment and retrieval by comparing the value of X for the intermediate scheme and the crawler scheme, on the one hand, with the conventional scheme, on the other, assuming that the tether is massless ($X = 1$). Assuming that the mass m_1 of the space station is large compared with the mass m_2 of the sub-satellite, we can define the quantity $X = \tilde{p}_f/p_f$ for the crawler scheme and intermediate scheme by the formulae

$$X \equiv X_S = \frac{3\mu_2 S_0^2 p_f^2 + \mu_3 L^2}{p_f^2(3\mu_2 S_0^2 + \mu_3 L^2)}, \quad X \equiv X_{LS} = \frac{3\mu_2 p_f^2 + \mu_3^0 K^2 q_f^3}{p_f^2(3\mu_2 + \mu_3^0 K^2)} \tag{4.5}$$

or

$$X_S - 1 = \frac{\mu_3 L^2(1 - p_f^2)}{p_f^2(3\mu_2 S_0^2 + \mu_3 L^2)}, \quad X_{LS} - 1 = \frac{\mu_3^0 K^2(q_f^3 - p_f^2)}{p_f^2(3\mu_2 + \mu_3^0 K^2)} \tag{4.6}$$

Hence, for the deployment procedure ($S^+/L^-, p_f > 1$) we obtain $X_S < 1$, and for the retrieval procedure ($S^-/L^+, p_f < 1$) we obtain $X_S > 1$. Thus, the behaviour of the system with the crawler scheme is better in both cases than its behaviour with the conventional scheme and a massless tether. For the intermediate scheme, in deployment ($S^+/L^-, p_f > 1, q_f < 1$) the condition $X_{LS} < 1$ is satisfied, and in retrieval (S^-/L^+, $p_f < 1, q_f > 1$), the condition $X_{LS} > 1$, that is, the behaviour of the system with the intermediate scheme is better in both deployment and retrieval than with the conventional scheme.

We will now compare the intermediate scheme with the crawler scheme:

$$X_{LS} - X_S = \frac{1+a}{1+b} - \frac{1+a_1}{1+b_1} \tag{4.7}$$

$$a = \frac{\mu_3^0 K^2 q_f^3}{3\mu_2 p_f^2}, \quad a_1 = \frac{\mu_3 L^2}{3\mu_2 S_0^2 p_f^2}, \quad b = \frac{\mu_3^0 K^2}{3\mu_2}, \quad b_1 = \frac{\mu_3 L^2}{3\mu_2 S_0^2}$$

For the deployment procedure ($S^+/L^-, p_f > 1, q_f < 1$), we will assume that the final length L_f of the tether in the intermediate scheme is the same as its constant length L in the crawler scheme, and that the initial positions S_0 of the sub-satellite are the same; then $a = a_1, b > b_1$.

Consequently, in deployment we have $X_{LS} < X_S$.

For the retrieval procedure ($S^-/L^+, p_f < 1, q_f > 1$), let us assume that the initial length L_0 of the tether in the intermediate scheme is the same as the length L of the tether in the crawler scheme, and that the values of S_0 are the same; then $b = b_1, a > a_1$.

Consequently, in retrieval we have $X_{LS} > X_S$.

Thus, in both cases, the behaviour of the system with the intermediate scheme is better than with the crawler scheme.

5. NUMERICAL RESULTS

A numerical investigation of a system with an intermediate deployment/retrieval scheme was carried out for the following parameter values: total mass of the system $m = 5 \times 10^3$ kg, mass of the sub-satellite $m_2 = 850$ kg, linear density of the tether $\rho = 7.5 \times 10^{-4}$ kg/m, deployment corresponds to $p_f = 10$ and retrieval corresponds to $p_f = 0.1$. For both procedures S^-/L^+ and S^+/L^-, the tether length q is assumed to vary exponentially.

$$q = \exp(K_l \nu), \quad K_l = \nu_f^{-1} \ln q_f$$

The form of L and S in retrieval S^-/L^+ as functions of the number N of orbital revolutions of the satellite mass centre is shown in Fig. 2. The upper curve is that of $L(t)$; the other curves, from top to bottom, represent $S_{\sin}(t)$, $S_{\text{tin}}(t)$, $S_{\exp}(t)$ and $S_{\text{hyp}}(t)$. In deployment S^+/L^-, the functions L and S are obtained by replacing t by $t_f - t$.

To compare the behaviour of systems with the usual conventional deployment/retrieval scheme, crawler scheme and intermediate scheme, we assume that in deployment of the tether the same final tether length is chosen in all three schemes, and in retrieval, the same initial tether length (see the table).

Equations (1.8) were integrated numerically over a time interval corresponding to 20 orbital revolutions of the satellite; it was shown that the nature of the oscillations is qualitatively the same in all three deployment/retrieval schemes.

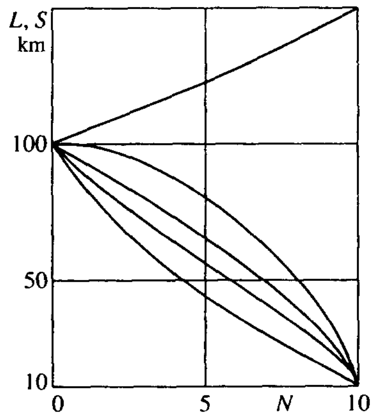

Fig. 2

	S_0, km	S_f, km	L_0, km	L_f, km	p_f	$\tilde{p}_f$
	Deployment of the system					
The conventional scheme	10	100			10	10
The crawler scheme	10	100	100	100	10	4.850
The intermediate scheme S^+/L^-	10	100	150	100	10	2.869
	Retrieval of the system					
The conventional scheme	100	10			0.1	0.1
The crawler scheme	100	10	100	100	0.1	0.2062
The intermediate scheme S^-/L^+	100	10	100	150	0.1	0.3485

As to the amplitudes of the oscillations that develop, the following observations are in order.

In deployment, the amplitude of the oscillations in the orbital plane of a system with the intermediate scheme is a few times less than for a system with the conventional and crawler schemes. The amplitude of the oscillations out of the orbital plane, however, is of the same order of magnitude for all three schemes.

In retrieval, the amplitudes of the final oscillations both in the orbital plane and out of it are considerably less for the intermediate scheme than for the other two schemes.

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